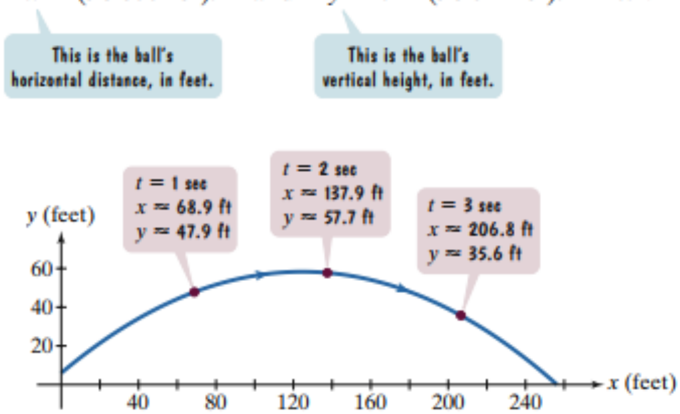
Period 4, April 10, 2025

Plane Curves and Parametric Equations

$$x = (90 \cos 40°)t \quad \text{and} \quad y = 6 + (90 \sin 40°)t - 16t^2.$$



Plane Curves and Parametric Equations

Suppose that t is a number in an interval I. A **plane curve** is the set of ordered pairs (x, y), where

$$x = f(t), \quad y = g(t) \quad \text{for } t \text{ in interval } I.$$

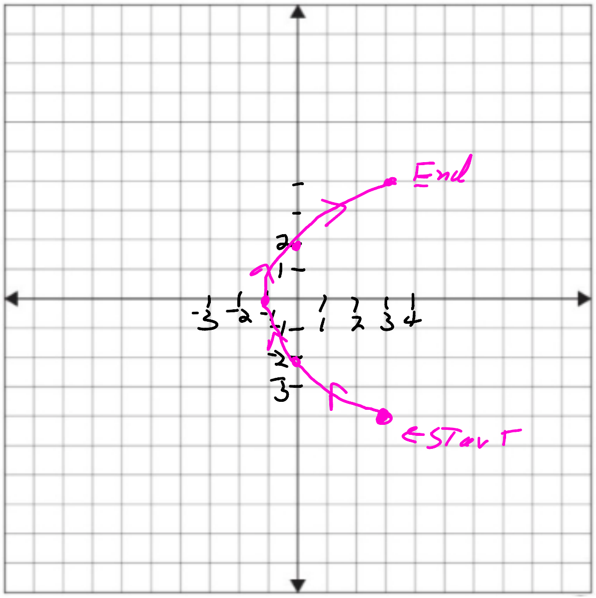
The variable t is called a **parameter**, and the equations $x = f(t)$ and $y = g(t)$ are called **parametric equations** for the curve.

Graphing a Plane Curve Described by Parametric Equations

1. Select some values of t on the given interval.
2. For each value of t, use the given parametric equations to compute x and y.
3. Plot the points (x, y) in the order of increasing t and connect them with a smooth curve.

Graph the plane curve defined by the parametric equations:

$$x = t^2 - 1, \qquad y = 2t, \qquad -2 \le t \le 2.$$



T	X (T^2-1)	Y ($2T$)
Start $\rightarrow$ -2	3	-4
-1	0	-2
0	-1	0
1	0	2
End $\rightarrow$ 2	3	4

$X = T^2 - 1 \Rightarrow X = \left(\frac{Y}{2}\right)^2 - 1 \Rightarrow X = \frac{1}{4}Y^2 - 1$

$Y = 2T \Rightarrow \frac{Y}{2} = T$

$(X-1)\cdot 4 = Y^2$

Sketch the plane curve represented by the parametric equations

$$x = \sqrt{t} \quad \text{and} \quad y = \tfrac{1}{2}t + 1$$

by eliminating the parameter. $X \ge 0$ $T \ge 0$

$X = \sqrt{T}$ $\qquad Y = \frac{1}{2}T + 1$

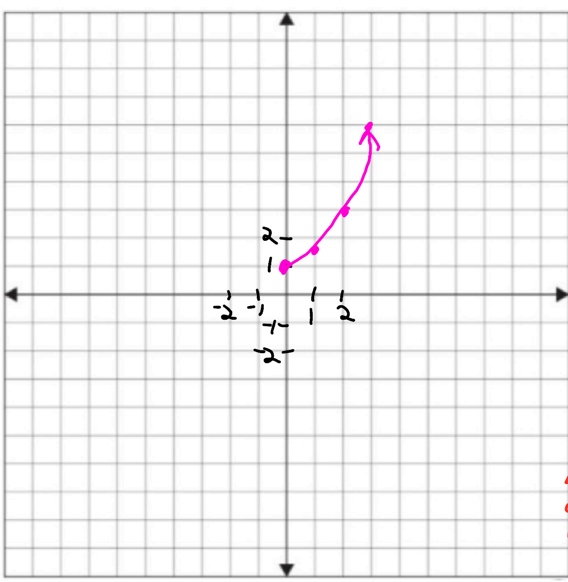
$X^2 = T$

$Y = \frac{1}{2}X^2 + 1 \Rightarrow Y - 1 = \frac{1}{2}X^2$

$4P(Y-K) = (X-h)^2$ $\qquad 2(Y-1) = X^2$ $\quad X \ge 0$

Vertex $(0,1)$

Focal Length $= \frac{1}{2}$



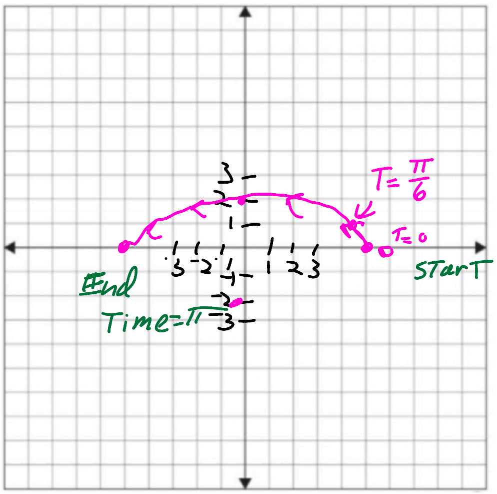
T	X	Y
0	0	1
1	1	$1\frac{1}{2}$
-1	$\varnothing$	$\varnothing$
4	2	3
9	3	$5\frac{1}{2}$

Sketch the plane curve represented by the parametric equations

$$x = 5\cos t, \quad y = 2\sin t, \quad 0 \le t \le \pi$$

by eliminating the parameter.

$\boxed{\begin{array}{l} 1+\tan^2 T = \sec^2 T \\ 1+\cot^2 T = \csc^2 T \end{array}}$



$X = 5\cos T$ $\qquad$ $Y = 2\sin T$

$\frac{X}{5} = \cos T$ $\qquad$ $\frac{Y}{2} = \sin T$

$\frac{X^2}{25} = \cos^2 T$ $\qquad$ $\frac{Y^2}{4} = \sin^2 T$

$\cos^2 T + \sin^2 T = 1$

$\frac{X^2}{25} + \frac{Y^2}{4} = 1$ Ellipse center $(0,0)$

$\sqrt{25} = 5 =$ semi major

$\sqrt{4} = 2 =$ semi minor

End $T = \pi$

$5\cos\pi, 2\sin\pi$

$(-5, 0)$

when Time $= 0$ $\quad (5, 0)$

Time $= \frac{\pi}{6}$ $\quad \left(\frac{5\sqrt{3}}{2}, 1\right)$

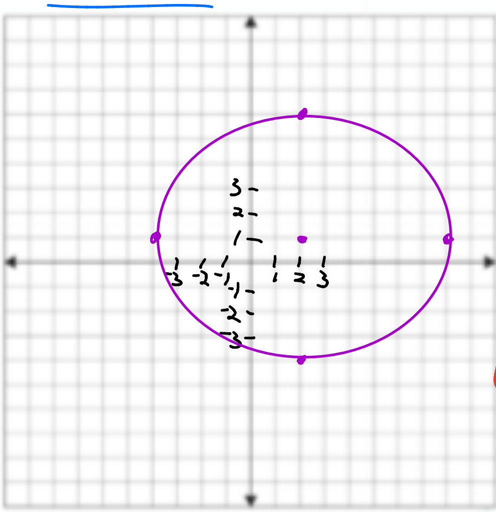
$(4.33, 1)$

Parametric Equations for the Function $y = f(x)$

One set of parametric equations for the plane curve defined by $y = f(x)$ is

$$x = t \quad \text{and} \quad y = f(t),$$

in which t is in the domain of f.



$\frac{(x-2)^2}{36} + \frac{(y-1)^2}{25} = 1$

Center $= (2, 1)$

$\sqrt{36} = 6$

$\sqrt{25} = 5$

$X = T$

$\frac{(T-2)^2}{36} + \frac{(y-1)^2}{25} = 1$

$(y-1)^2 = 25\left(1 - \frac{(T-2)^2}{36}\right)$

$y = 1 + \sqrt{25\left(1 - \frac{(T-2)^2}{36}\right)}$

$X = T^2 + 1$

$\frac{((T^2+1)-2)^2}{36} + \frac{(y-1)^2}{25} = 1$

Clean up

Find the focus and directrix of the parabola with the equation $24x^2 + 4y = 0$. Then, graph the parabola.

$-4y \quad -4y$

$$x^2 = 4py$$

$$y^2 = 4px$$

(h,k)

vertex

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

The focus is $\left(0, -\frac{1}{24}\right)$.

(Simplify your answer. Type an ordered pair.)

The directrix is $y = \frac{1}{24}$.

(Simplify your answer.)

Use the graphing tool to graph the parabola only.

$$\frac{24x^2}{24} = \frac{-4y}{24}$$

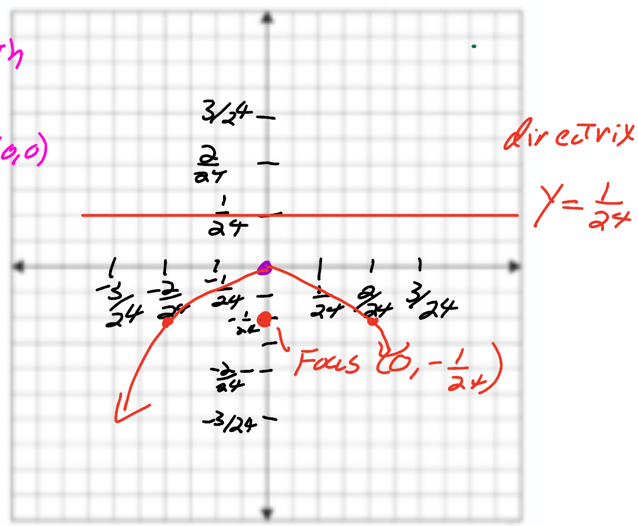
$$x^2 = -\frac{4}{24}(y)$$

$$x^2 = -4\left(\frac{1}{24}\right)(y)$$

Focal Length

vertex (0,0)

opens Down



Find the standard form of the equation of the parabola satisfying the given conditions.

Focus: $(-5,4)$; Directrix: $y = 2$

The standard form of the equation is $\square$.

(Type an equation. Simplify your answer.)

$(x-h)^2 = 4p(y-k)$

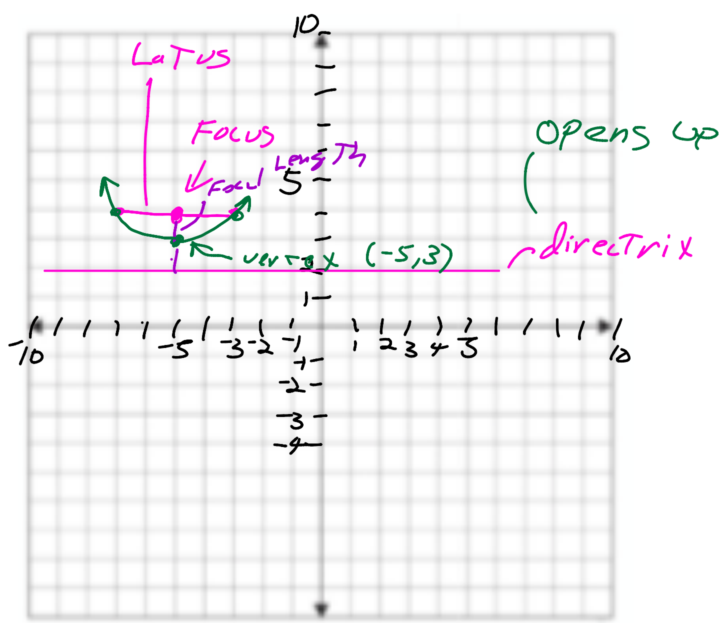
$h = -5$

$k = 3$

Focal Length $= p = 1$

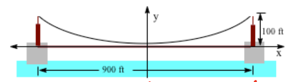
$(x-(-5))^2 = 4 \cdot 1(y-3)$

$(x+5)^2 = 4(y-3)$



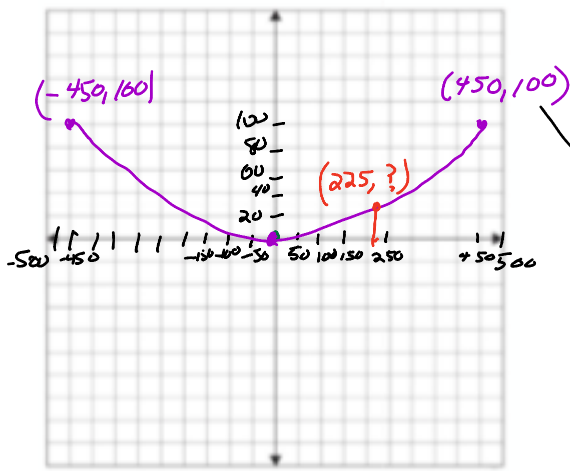
The cables of a suspension bridge are in the shape of a parabola, as shown in the figure. The towers supporting the cable are 900 feet apart and 100 feet high. If the cables touch the road surface midway between the towers, what is the height of the cable at a point 225 feet from the center of the bridge?

$X = 225$



The cable is ☐ feet from the road surface at a point 225 feet from the center of the bridge.
(Simplify your anwer.)

opens up
vertex $= (0, 0)$



$$x^2 = 4py$$
$$(450)^2 = 4p(100)$$
$$202500 = 400p$$
$$506.25 = p$$
$$x^2 = 4(506.25) \cdot y$$
$$x^2 = 2025y$$
$$(225)^2 = 2025y$$
$$\frac{50625}{2025} = \frac{2025y}{2025} \Rightarrow 25 = y$$

Find the standard form of the equation of the parabola satisfying the given conditions.

Vertex: (6, − 1); Focus: (6, − 3)

The standard form of the equation is $(x-6)^2 = -8(y+1)$.
(Type an equation. Simplify your answer.)

Focal length $= p = 2$

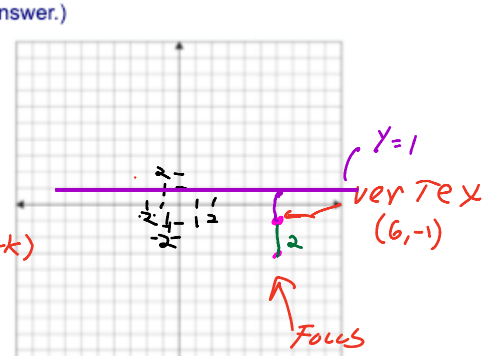
directrix up 2 from vertex

opens down

$(x-h)^2 = -4p(y-k)$

$(h, k) \Rightarrow (6, -1)$

$$(x-6)^2 = -4 \cdot 2(y-(-1))$$
$$(x-6)^2 = -8(y+1)$$



Find the solution set for the system by graphing both of the system's equations in the same rectangular coordinate system and finding points of intersection. Check all solutions in both equations.

$$\begin{cases} x = (y+4)^2 - 1 \Rightarrow X+1 = (y+4)^2 \text{ Parabola opens Right} \\ (x-4)^2 + (y+4)^2 = 1 \Rightarrow \text{Circle center}(4,-4) \text{ radius} = 1 \end{cases}$$

$1(X+1) = (y+4)^2$

Vertex $= (-1, -4)$

Focal Length $= \frac{1}{4}$

$4P = 1$

$P = \frac{1}{4}$

Use the graphing tool to graph the system.

Find the solution set for the system. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. { } (Type an ordered pair. Use a comma to separate answers as needed.)

B. The solution set is ∅.

$x = (y+4)^2 - 1$

X	Y
3	-2
0	-3
0	-5
3	-6